GAIA AND THE HUNT FOR EXTRA-SOLAR PLANETS

M.G. Lattanzi,[1] **A. Spagna,**[1] **A. Sozzetti,**[2] **S. Casertano,**[3*]

[1]Osservatorio Astronomico di Torino, Pino Torinese TO, I-10025, Italy
[2]Dipartimento di Fisica Generale, Universitá di Torino, I-10100, Italy
[3]Space Telescope Science Institute, Baltimore MD, 21218, USA

ABSTRACT

We present the results of realistic end-to-end simulations of observations of nearby stars with the proposed global astrometry mission GAIA, recently recommended within the context of ESA's Horizon 2000 Plus long-term scientific program. We show that under realistic, if challenging, assumptions, GAIA will be capable of surveying the solar neighborhood within 100-200 pc for the astrometric signatures of planets around stars down to $V = 16$ mag.

The wealth of results on the frequency and properties of massive planets from GAIA observations will provide a formidable testing ground on which to confront the most sophisticated theories on planetary formation and evolution.

Finally, we suggest the possibility of more sophisticated probabilistic detection techniques which may be able to detect the presence of Earth-like planets around stars within 20 pc.

Key words: astrometry; stars:planetary systems; GAIA.

1. INTRODUCTION

After many years of work, radial velocity searches have finally succeeded in finding a number of Jupiter-like planets orbiting nearby stars. Currently, there exist about 10 candidate planets, all found by the radial velocity technique around stars of solar type or later (see e.g. Mayor & Queloz 1995, Marcy & Butler 1996, Butler & Marcy 1996).

However, most candidate planetary systems identified thus far seem to defy the prior expectations of well-established theoretical explanations of the formation and evolution of planets. Giant planets appear to have either very short periods (Mayor & Queloz 1995), placing them well outside the freezing zone where their formation was expected to take place, or have high eccentricity (Cochran et al 1997),

again in contrast with the planetesimal accretion model.

Astrometric techniques are complementary to the radial velocity detection currently employed, in that, for given mass, the sensitivity of astrometry and radial velocity techniques respectively increases and decreases with period. Thus, astrometric searches will find preferentially planets at several AU from the central star, with periods of several years, as opposed to radial velocity searches which favor planets very close to the central star.

A GAIA-like satellite, capable of extremely high precision astrometric measurements, will revolutionize our knowledge of planetary systems and provide invaluable statistical information for theoretical formation models. With the baseline properties currently envisioned (Lindegren & Perryman 1996), GAIA will find more than 50 per cent of all Jupiter-like planets orbiting stars within 100 pc with periods between 1 and 15 years, averaged over eccentricity, inclination, and orbital phase, as well as lower-mass planets around more nearby stars. The orbital elements of many detected systems can be evaluated, including mass and orbital radius, unlike radial velocity detection which leaves significant uncertainty in the two quantities because of the inclination uncertainty. If solar-like planetary systems are reasonably common, we can expect hundreds of thousands of detections, providing ample material to test both theoretical predictions and detailed models. Even detection of Earth-like planets could become viable in a statistical sense, under appropriate circumstances.

2. DATA SIMULATION

The simulation code is adapted from that used by Galligani et al (1989) for the assessment of the astrometric accuracy of the sphere reconstruction in the Hipparcos mission. We generate catalogs of single stars randomly distributed on the sky; each run produces a sphere of N stars, with the same value of parallax, total proper motion, magnitude and color.

The simulations presented here are carried out within the great circle approximation (Lindegren & Kovalevski 1989). We neglect all difficulties related to the reconstruction and calibration of individual great

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*Affiliated to the Space Sciences Dept., ESA

Table 1. Photon and total error for a single observation (10 elementary exposures)

V mag	Photon error (μas)	Total error (μas)	
		($\sigma_b = 200$ pm)	($\sigma_b = 20$ pm)
0	0.032	5.3	0.53
1	0.050	5.3	0.53
2	0.080	5.3	0.54
3	0.126	5.3	0.55
4	0.20	5.3	0.57
5	0.32	5.3	0.62
6	0.50	5.3	0.73
7	0.80	5.4	0.96
8	1.3	5.5	1.4
9	2.0	5.7	2.1
10	3.2	6.2	3.2
11	5.0	7.3	5.1
12	8.0	9.6	8.0
13	12.6	13.7	12.6
14	20.0	20.7	20.0
15	31.7	32.2	31.8
16	50.3	50.6	50.3
17	79.7	79.9	79.7
18	126.4	126.5	126.4
19	200.3	200.4	200.3
20	317.5	317.5	317.5

circles, difficulties that would be more properly addressed within the context of a global model of the Galaxy, and consider as the *basic observable* the abscissa ψ along the instantaneous great circle measured in each observation of each star. The accuracy expected for individual measurements of ψ has been discussed in Casertano et al (1996), whose basic assumptions we adopt here. The measurement error expected for individual observations depends on the instrumental parameters, here taken as in Lindegren & Perryman (1996), on the magnitude of the star and on the quality of the metrology control; the results of Casertano et al (1996) are reported in Table 1 for reference.

The new element added here is that the instantaneous "true" position of each star includes the gravitational perturbation (Keplerian motion) induced by a single, non-luminous planetary mass orbiting the star.

3. DETECTION OF JUPITER-LIKE PLANETS

3.1. Star luminosity and measurement error

We consider first the case of planets with mass comparable to that of Jupiter. Such planets produce a relatively large astrometric perturbation; at 10 pc the reflex motion of the Sun due to Jupiter's motion would be 500 μas with a period of 11.8 years. We will parametrize the detection probability by the two major contributors, the period P and the signature

$$\alpha = \frac{M_p}{M_s} \frac{a_p}{D} \qquad (1)$$

where M_p, M_s are the masses of the planet and star respectively, a_p the semimajor axis of the planetary system, and D its distance from us. If a_p is in

AU, and D in parsec, then α is expressed in arcseconds. We generally assume a single-observation measurement error $\sigma_\psi = 10$ μas, appropriate to a star brighter than $V \sim 12$ mag (see Table 1), corresponding to the Sun at 200 pc.

We also carried out tests with different measurement errors, and demonstrated explicitly that the detection probability depends exclusively on the "signal-to-noise" ratio

$$S/N = \alpha/\sigma_\psi$$

so that rescaling to different measurement errors is straightforward.

3.2. Detection method

Our detection method for Jupiter-like planets is a classical application of the χ^2 test. After assigning each star in the simulation a secondary component, we solve for the five astrometric parameters for that star as if it had no companion. We then apply a standard χ^2 test (with the confidence level set to 95 per cent) to the residuals $\psi - \psi_r$, where the ψ are the actual measurements, and the ψ_r the great circle abscissae recomputed on the basis of the single-star fit. If the test is failed, that is, the residuals are significant at the 95 per cent level, the planet is considered "detected". Note that this method measures only deviations from the single-star model and makes no assumptions on the nature of the residuals, nor does it give an indication of whether the planet's parameters can be computed.

The simulated data for this case include a sample of 160 000 stars uniformly distributed over the sky. Each star is assumed to have a planet inducing a reflex astrometric motion of amplitude α ranging from 5 to 100 μas, and period P between 0.5 and 20 years. The remaining orbital elements are distributed randomly in the ranges: $0° \leq i \leq 90°$, $0 \leq e \leq 0.3$, $0 \leq \Omega \leq 2\pi$, $0 \leq \omega \leq 2\pi$, $0 \leq T \leq P$.

This means that the detection probability discussed in this study must be considered as averaged on both the mission parameters (such as number of observations vs ecliptic latitude) and on the orbital elements (e.g. inclination and eccentricity). Detailed investigations of these detection methods and their dependence on those parameters are in progress and will be presented elsewhere.

Finally, since the "detection" of a planet is indicated by a χ^2 deviation significant at the 95 per cent level, we would expect a 5 per cent incidence of false detections. As a check, we repeated the simulation without planets, and we did in fact find false detections consistent with the expected 5 per cent.

3.3. Results

The fraction of planets detected—as measured by the failure of the χ^2 test for the single-star hypothesis—is given in Figure 1 as a function of orbital period and α, for an assumed measurement error of 10μas. We note that at relatively low S/N ratios the detection
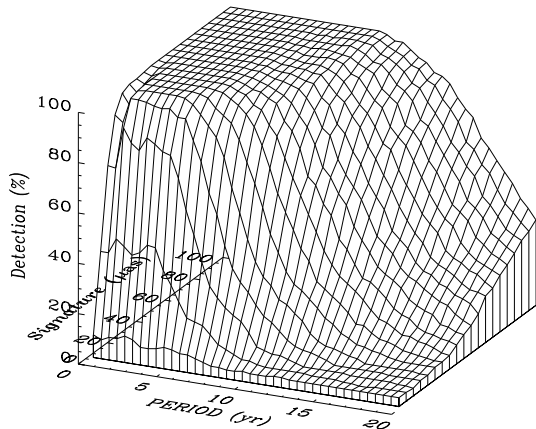
Figure 1. Planet detection probability as function of the astrometric signature α and of the orbital period P, for $\sigma_\psi = 10$ μas. The percentage of detection of each point is based on 200 random planetary systems uniformly distributed on the sky.
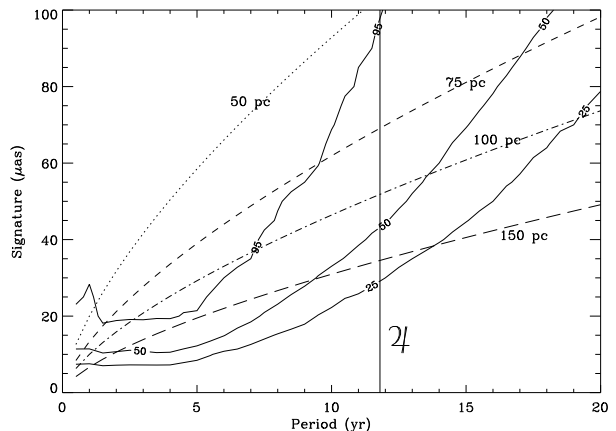

Figure 2. Iso-probability contours (solid lines) for 25%, 50% and 95% of detection probability, compared with Kepler's third laws (dotted/dashed lines) for systems with Jupiter-Sun masses at D = 50, 75, 100 and 150 pc. Jupiter-like planets ($P = 11.8$ yr) appear detectable, with probability $\geq$ 50%, up to a distance of 100 pc (vertical line).

probability is dominated by sampling of the orbital period, while at higher S/N values orbital sampling is less critical and long period planets (up to about twice the mission duration) are detectable. For instance, the detection probability reaches about cent per cent when S/N $\rightarrow$ 10.

Figure 1 also shows that for S/N$\rightarrow$ 1 the χ^2 test quickly loses its sensitivity. The shallow dip in the detection probability at $P \sim 1$ year is the result of the coupling between orbital and parallactic motion.

As stated above, the results shown in Figure 1 can be scaled easily to other measurement accuracies, whether due to different assumptions on the properties of the mission or to different stellar magnitudes; for example, for measurement accuracy $\sigma_\psi = 1\mu$as, the detection probability is exactly the same as shown in the Figure 1, but for an amplitude ten times smaller—thus maintaining the same S/N ratio.

3.4. Parameters for 50 per cent detection

Another way to look at these results is to determine the amplitude of the perturbation needed for a certain probability of detection, as a function of the planet's period, and compare this relation with the Kepler's third law, $\alpha \propto P^{2/3}$, which depends explicitly on the physical parameters of the planetary system.

The empirical relations derived from Figure 1 are shown in Figure 2 for three levels of detection efficiency (25, 50, 95 per cent). Orbital periods shorter than 5 years are well-matched to the mission length and sampling law, and therefore the detection probability is nearly independent of orbital period, with a detection probability of 50 per cent when $S/N \sim 1$. On the other hand, if the period exceeds the mission lifetime, the probability of planet detection drops significantly, and a much higher signal is required for the

planet's signature to be detected. This is in qualitative agreement with the results of Babcock (1994), who studied the detection *and* convergence probability of a complete orbital model for simulated planetary systems as observed by the mission POINTS. Babcock (1994) did find a slightly larger sensitivity on planet period, manifested in an earlier turn-up and steeper slope at long periods of the 50 per cent probability curve; this most likely depends on the fact that the determination of reliable orbital elements is more challenging than detection only.

A planet with exactly the same characteristics as Jupiter ($P = 11.8$ yr) can be detected with probability greater than 50 per cent to a distance of 100 pc, while planets with mass similar to Jupiter's, but shorter orbital periods, can be detected much further away, to distances exceeding 150 pc for periods between 2 and 9 yr. Depending on the orbital period, the number of candidate stars for detection of Jupiter-like planets may well be several hundred thousand (see also Casertano et al 1995).

4. DETECTABILITY OF KNOWN CANDIDATE PLANETS

As a further test of GAIA's capabilities, we consider explicitly three candidate planets discovered by the radial velocity technique, 47 Uma, 70 Vir, and 51 Peg. The parent stars of all three systems are close by ($d \leq 20$ pc) and very similar to our Sun. According to the spectroscopic measurements summarized in Perryman et al. (1996), the planets orbiting these stars have minimum masses of ~ 2.46, ~ 6.50, and $\sim 0.5M_{\rm J}$, and orbital periods of about 3 years, 4 months, and 4 days, respectively. This translates in the following minimum astrometric signatures:

$$\alpha_{47\text{UMa}} \geq 362 \ \mu\text{as}$$
$$\alpha_{70\text{Vir}} \geq 168 \ \mu\text{as}$$
$$\alpha_{51\text{Peg}} \geq 1.66 \ \mu\text{as}$$

The astrometric detection of 51 Peg by a GAIA-like mission is extremely difficult, because of the small astrometric signature (short period implies small separation, thus small reflex motion) and of the mismatch between the orbital period and the frequency of GAIA observations. On the other hand, the detection of the signatures induced on 47 UMa and 70 Vir should be a much easier task for a GAIA-like satellite.

In the context of a new class of simulations, we have generated 100 planetary systems on the celestial sphere, respectively identical to 47 UMa and 70 Vir, assuming the stars to be of mass $M = M_\odot$, and assuming perfectly circular orbits. The inclination of the orbital planes, undetermined parameter in the case of radial velocity measurements, was initially chosen to be $i = 45°$.

The χ^2 test indicated a detection probability of essentially 100 per cent for these planets for a single-observation error of $\sigma_\psi = 10\mu$as (a conservative assumption for these relatively bright stars). We were also able to recover accurately all orbital elements and to reconstruct the apparent path of the stars on the plane of the sky. The details of these simulations will be published elsewhere.

5.   DETECTION OF EARTH-LIKE PLANETS

Even for very nearby stars, within 10 pc or so, detection of Earth-like planets will be extremely challenging. The astrometric signature of the Earth on the position of the Sun corresponds to about 0.3 μas at 10 pc, beyond the capabilities currently projected for GAIA.

The question we address here is whether it may be possible to establish *statistically* the presence of Earth-like planets, even though they cannot be detected directly on an individual basis. Specifically, we consider the possibility that the statistical properties of the residuals for a few hundred stars might bear a weak signature of the presence of Earth-like planets, and that, by combining these data, the evidence for the presence of such planets might be uncovered.

To this end, we use the least-squares technique in a non-conventional way. Contrary to what we have done before, we *assume* the presence of a planet, and fit the observations with a model which includes the semi-major axis, a, of the stellar orbital motion along with the five astrometric parameters. We repeat this process for different values of the period P. Convergence of the fit, and not the significance of the values derived for a, is taken as the indicator of a positive detection.[1] Observations for Earth-Sun systems were simulated with an error $\sigma_\psi = 1 \ \mu$as as indicated in Table 1 for nearby solar stars, in the case of a metrology accuracy of 20 pm. As for the case of the χ^2

test, the same simulations were repeated without astrometric signatures in order to assess the probability of false detections.

We find that the probability of convergence does in fact depend on the presence of the planet. As shown in Figure 3 for systems at 20 parsec, fitting a system without a planet or with the incorrect period results in a smaller convergence probability than if the planet is present and the period correct. For an initial value $a_0 = 0.34\mu$as, the fit converges only 25 per cent of the time if there is no planet or if the planet has a period different by as little as 0.05 yr from the period used in the fit. On the other hand, if a planet with the correct period is present, the fit converges about 45 per cent of the time. This indicates that a statistical signature of the presence of an Earth-like planet is indeed present, albeit weak. Of course, this cannot lead to the actual detection of individual planets. However, given a sufficient number of candidates—there are about 400 eligible stars within 20 pc—this method might be used to detect whether, and how often, Earth-like planets may be present, even though the S/N of the signature of individual planets may be insufficient for a more formal detection. Of course, there will be little or no information on the detailed orbital parameters of such planets, but the interest, scientific and not, of the detection of Earth-like planets is such that even probabilistic detections are desirable.

The current analysis is still too simplistic to assess whether this method, or variants thereof, can be successful. We need to investigate further the effect of the other parameters—initial phase, inclination, eccentricity, etc.—which for now have been assumed "known"; initial phase may have an especially large effect on the convergence of the fit. Similarly, a method based on the statistical analysis of low S/N cases will only be successful if the error properties of the measurements are extremely well-known, and if the possible presence of other planets does not affect the convergence of the fit. On the other hand, the ability to "detect" planets with $S/N < 1$ is very tantalizing, and well worth of further study.

6.   SUMMARY AND CONCLUSIONS

Detailed simulations of observation of star-planet systems within the baseline framework of the proposed GAIA mission indicate the probable discovery of a very large number of massive, Jupiter-like planets around stars as far away as 100-200 pc, thus enabling a qualitative jump in the statistical study of planetary systems and new understanding of their formation. Specific simulations of known candidate planetary systems, discovered by the radial velocity technique, indicate that such systems will be easy to discover and their orbital parameters will be determined accurately by GAIA, with the exception of very short-period systems such as 51 Pegasi.

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[1] As expected, because of the low S/N ratios, the error of the estimated values of a is usually large and not very significant.
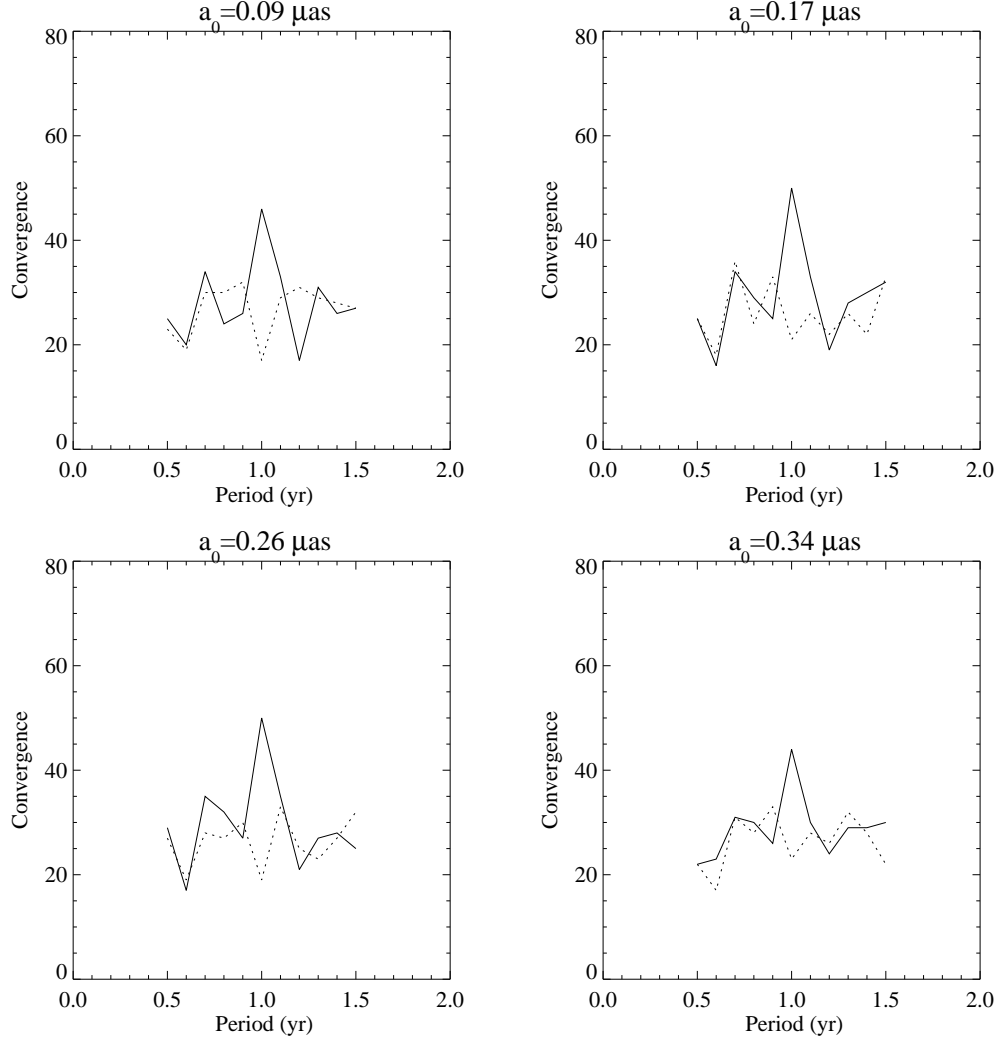
Figure 3. Convergence probabilities for Sun-Earth planetary systems (solid curve) at 20 pc as function of the initial guesses on the one-year period. The dashed curve shows the number of false detection. Panels refer to different starting values of the orbital radius a_0; the correct value is $a = 0.17$ μas. The 22 data points used to plot both graphs are based on 100 stars each.

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